

It is indeed a fundamental construction of all linear codes

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Received: date / Accepted: date

Abstract Linear codes are widely employed in communication systems, consumer electronics, and storage devices. All linear codes over finite fields can be generated by a generator matrix. Due to this, the generator matrix approach is called a fundamental construction of linear codes. This is the only known construction method that can produce all linear codes over finite fields. Recently, a defining-set construction of linear codes over finite fields has attracted a lot of attention, and have been employed to produce a huge number of classes of linear codes over finite fields. It was claimed that this approach can also generate all linear codes over finite fields. But so far, no proof of this claim is given in the literature. The objective of this paper is to prove this claim, and confirm that the defining-set approach is indeed a fundamental approach to constructing all linear codes over finite fields. As a byproduct, a trace representation of all linear codes over finite fields is presented.

Keywords Cyclic codes · linear codes · weight distribution · weight enumerator · trace function

1 Introduction

Throughout this paper, let q be a power of a prime p . An $[n, k, d]$ linear code C over $\text{GF}(q)$ is a k -dimensional subspace of $\text{GF}(q)^n$ with minimum Hamming distance d . Let A_i denote the number of codewords with Hamming weight i in a linear code C of length n . The *weight enumerator* of C is defined by

$$1 + A_1z + A_2z^2 + \cdots + A_nz^n.$$

The *weight distribution* of C is the sequence $(1, A_1, \dots, A_n)$.

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An $[n, k]$ linear code C over $\text{GF}(q)$ is called *cyclic* if $(c_0, c_1, \dots, c_{n-1}) \in C$ implies $(c_{n-1}, c_0, c_1, \dots, c_{n-2}) \in C$. We can identify a vector $(c_0, c_1, \dots, c_{n-1}) \in \text{GF}(q)^n$ with

$$c_0 + c_1x + c_2x^2 + \dots + c_{n-1}x^{n-1} \in \text{GF}(q)[x]/(x^n - 1).$$

In this way, a code C of length n over $\text{GF}(q)$ corresponds to a subset of the quotient ring $\text{GF}(q)[x]/(x^n - 1)$. A linear code C is cyclic if and only if the corresponding subset in $\text{GF}(q)[x]/(x^n - 1)$ is an ideal of the ring $\text{GF}(q)[x]/(x^n - 1)$.

It is well-known that every ideal of $\text{GF}(q)[x]/(x^n - 1)$ is principal. Let $C = \langle g(x) \rangle$ be a cyclic code, where $g(x)$ is monic and has the smallest degree among all the generators of C . Then $g(x)$ is unique and called the *generator polynomial*, and $h(x) = (x^n - 1)/g(x)$ is referred to as the *check polynomial* of C .

Cyclic codes over finite fields can be generated by a generator matrix, or a generator polynomial, or a generating idempotent. Under certain conditions, cyclic codes over finite fields have a trace representation described in the following theorem whose proof is based on Delsarte's Theorem [18, 2].

Theorem 1 (Wolfmann) *Let C be a cyclic code of length n over $\text{GF}(q)$ with parity-check polynomial $h(x)$, where $\gcd(n, q) = 1$. Let β be a primitive n -th root of unity over $\text{GF}(q^m)$, where $m := \text{ord}_n(q)$ is the order of q modulo n . Let J be a subset of $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ such that*

$$h^*(x) = \prod_{j \in J} m_{\beta^j}(x),$$

where $m_{\beta^j}(x)$ denotes the minimal polynomial of β^j over $\text{GF}(q)$, and $h^(x)$ is the reciprocal of $h(x)$. Then C consists of all the following codewords*

$$c_a(x) = \sum_{i=0}^{n-1} \text{Tr}(f_a(\beta^i))x^i,$$

where Tr denotes the trace function from $\text{GF}(q^m)$ to $\text{GF}(q)$, and

$$f_a(x) = \sum_{i \in J} a_i x^i, \quad a_i \in \text{GF}(q^m).$$

This trace representation of certain sub-classes of cyclic codes was known for a long time. For example, the trace representation of irreducible codes dates back at least to Baumert and McEliece [1]. A trace description of irreducible quasi-cyclic codes was given in [17]. The trace representation of cyclic codes over finite fields in Theorem 1 was presented and proved by Wolfmann in [18] under the restriction that $\gcd(q, n) = 1$. It demonstrates another way of generating many cyclic codes over finite fields. The importance of this trace representation is mostly demonstrated by its application in determining the weight distribution (also called the weight enumerator) of cyclic codes over finite fields. The trace representation allows one to determine the weight distribution of a cyclic code by evaluating certain types of character sums over finite fields, and has led to a lot of recent progress on the weight distribution problem of cyclic codes [5, 9].

However, to the best knowledge of the author, no similar trace representation of linear codes over finite fields is available in the literature. Even the trace representation in Theorem 1 has the restriction that $\gcd(n, q) = 1$ and thus applies to only a special type of cyclic codes. One objective of this paper is to give a trace representation of all linear codes over finite fields.

It is well known that all linear codes over finite fields can be generated with a generator matrix. Because of this fact, the generator matrix approach is a fundamental approach to constructing all linear codes over finite fields and is the only one. Recently, a defining-set construction of linear codes over finite fields has been intensively investigated, and shown to be a promising approach, as many classes of linear codes with good parameters have been produced. It was claimed in [3] that all linear codes over finite fields can be produced with this approach. But so far, no proof has been seen in the literature. Another objective of this paper is to provide this claim, and confirm that it is indeed a fundamental construction of all linear codes over finite fields.

2 A generic construction of linear codes over finite fields

Throughout this section, let q be a prime power and let $r = q^m$, where m is a positive integer. Let Tr denote the trace function from $\text{GF}(r)$ to $\text{GF}(q)$ unless otherwise stated.

2.1 The description of the construction

Let $D = \{d_1, d_2, \dots, d_n\} \subseteq \text{GF}(r)$. We define a code of length n over $\text{GF}(q)$ by

$$C_D = \{(\text{Tr}(xd_1), \text{Tr}(xd_2), \dots, \text{Tr}(xd_n)) : x \in \text{GF}(r)\}, \quad (1)$$

and call D the *defining set* of this code C_D . Since the trace function is linear, the code C_D is linear. By definition, the dimension of the code C_D is at most m .

Different orderings of the elements of D give different linear codes, which are however permutation equivalent. Hence, in this paper, we do not distinguish these codes obtained by different orderings, and do not consider the ordering of the elements in D . It should be noticed that the defining set D could be a multiset, i.e., some elements in D may be the same.

2.2 The generator matrix of the trace code C_D

Every linear code over a finite field must have a generator matrix. In this subsection, we derive a generator matrix for the trace code C_D , where

$$D = \{d_1, d_2, \dots, d_n\} \subseteq \text{GF}(r). \quad (2)$$

Let $\{\alpha_1, \alpha_2, \dots, \alpha_m\}$ be a basis of $\text{GF}(r)$ over $\text{GF}(q)$, and $\{\beta_1, \beta_2, \dots, \beta_m\}$ be its dual basis. By the definition of the dual basis,

$$\text{Tr}(\alpha_i \beta_j) = \begin{cases} 0 & \text{for } i \neq j, \\ 1 & \text{for } i = j. \end{cases} \quad (3)$$

Note that every basis of $\text{GF}(q^m)$ over $\text{GF}(q)$ has its dual basis [16, p. 58].

Let

$$d_i = \sum_{j=1}^m d_{j,i} \alpha_j, \text{ where all } d_{j,i} \in \text{GF}(q)$$

and

$$x = \sum_{h=1}^m x_h \beta_h, \text{ where all } x_h \in \text{GF}(q).$$

Then we have

$$\text{Tr}(d_i x) = \text{Tr} \left(\sum_{h=1}^m \sum_{j=1}^m x_h d_{j,i} \text{Tr}(\alpha_j \beta_h) \right) = \sum_{h=1}^m x_h d_{h,i}. \quad (4)$$

Consequently,

$$\mathbf{c}_x = (\text{Tr}(d_1 x), \text{Tr}(d_2 x), \dots, \text{Tr}(d_n x)) = (x_1, x_2, \dots, x_n) \mathbf{D} \quad (5)$$

where

$$\mathbf{D} = \begin{bmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,n} \\ d_{2,1} & d_{2,2} & \cdots & d_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m,1} & d_{m,2} & \cdots & d_{m,n} \end{bmatrix}. \quad (6)$$

As a result, \mathbf{D} is a generator matrix of the code \mathcal{C}_D , and depends on the choice of the basis $\{\alpha_1, \alpha_2, \dots, \alpha_m\}$. We will need this generator matrix later in this paper.

2.3 Weights in the codes \mathcal{C}_D

Define for each $x \in \text{GF}(r)$,

$$\mathbf{c}_x = (\text{Tr}(x d_1), \text{Tr}(x d_2), \dots, \text{Tr}(x d_n)), \quad (7)$$

The Hamming weight $\text{wt}(\mathbf{c}_x)$ of \mathbf{c}_x is $n - N_x(0)$, where

$$N_x(0) = |\{1 \leq i \leq n : \text{Tr}(x d_i) = 0\}|$$

for each $x \in \text{GF}(r)$.

It is easily seen that for any $D = \{d_1, d_2, \dots, d_n\} \subseteq \text{GF}(r)$ we have

$$\begin{aligned} q N_x(0) &= \sum_{i=1}^n \sum_{y \in \text{GF}(q)} \tilde{\chi}_1(y \text{Tr}(x d_i)) \\ &= \sum_{i=1}^n \sum_{y \in \text{GF}(q)} \chi_1(y x d_i) \\ &= n + \sum_{i=1}^n \sum_{y \in \text{GF}(q)^*} \chi_1(y x d_i) \\ &= n + \sum_{y \in \text{GF}(q)^*} \chi_1(y x D), \end{aligned} \quad (8)$$

where χ_1 and $\tilde{\chi}_1$ are the canonical additive characters of $\text{GF}(r)$ and $\text{GF}(q)$, respectively, aD denotes the set $\{ad : d \in D\}$, and $\chi_1(S) := \sum_{x \in S} \chi_1(x)$ for any subset S of $\text{GF}(r)$. Hence,

$$\text{wt}(\mathbf{c}_x) = n - N_x(0) = \frac{(q-1)n - \sum_{y \in \text{GF}(q)^*} \chi_1(yxD)}{q}. \quad (9)$$

Thus, the computation of the weight distribution of the code C_D reduces to the determination of the value distribution of the character sum

$$\sum_{y \in \text{GF}(q)^*} \sum_{i=1}^n \chi_1(yx d_i).$$

2.4 Comments on the trace construction and the objective of this paper

This construction technique has a long history [1], and was employed in [8], [7] and [3] for obtaining linear codes with a few weights. Recently, this trace construction of linear codes has attracted a lot of attention, and a huge amount of linear codes with good parameters are obtained in [4, 10, 11, 12, 13, 19, 20, 21].

It was claimed in [3] that every linear code C over a finite field $\text{GF}(q)$ has a trace construction, i.e., $C = C_D$ for some defining set $D \in \text{GF}(q^m)$, where m is a positive integer. However, to the best of the author's knowledge, no proof of this claim is available in the literature. We will prove this statement shortly.

3 A trace representation of linear codes over finite fields

In this section, we will give a trace representation of all linear codes over finite fields. Specifically, we will prove that any linear code C of length n over $\text{GF}(q)$ can be expressed as a trace code C_D , where D is a subset of $\text{GF}(q^m)$ for some positive integer m .

Let

$$\mathbf{D}' = \begin{bmatrix} d'_{1,1} & d'_{1,2} & \cdots & d'_{1,n} \\ d'_{2,1} & d'_{2,2} & \cdots & d'_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ d'_{k,1} & d'_{k,2} & \cdots & d'_{k,n} \end{bmatrix}. \quad (10)$$

be any matrix whose row vectors span a linear code C of length n over $\text{GF}(q)$, where k is equal to or more than the dimension of C .

Set $m = \max\{k, \lceil \log_q n \rceil\}$. By definition, we have $m \geq k$ and $n \leq q^m$. Now put

$$d'_i = \sum_{j=1}^k d'_{j,i} \alpha_j \text{ for all } 1 \leq i \leq n,$$

where $\{\alpha_1, \alpha_2, \dots, \alpha_m\}$ is a basis of $\text{GF}(q^m)$ over $\text{GF}(q)$.

Define $D' = \{d'_1, d'_2, \dots, d'_n\}$. It then follows from the discussion in Section 2.2 that the trace code $C_{D'}$ has generator matrix \mathbf{D}' , and is thus equal to C .

In general, we would have m as small as possible when we wish to have a trace construction of a linear code C . To this end, we may have a generator matrix whose row vectors are linearly independent over $\text{GF}(q)$, i.e., the parameter k above is equal to the dimension of C . It is now clear that a linear code over a finite field has many trace constructions (representations), depending on the choice of a generator matrix.

Example 1 Let $q = 2$ and $n = 7$. Let C be the binary code with length $n = 7$ and generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}. \quad (11)$$

Then C has dimension $k = 3$. Let $m = \max\{k, \lceil \log_q n \rceil\} = 3$. Let α be a generator of $\text{GF}(2^3)^*$ with $\alpha^3 + \alpha + 1 = 0$. Define $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$, where

$$d_1 = 1, d_2 = \alpha, d_3 = \alpha^2, d_4 = 1, d_5 = \alpha^3, d_6 = \alpha^4, d_7 = \alpha^2.$$

Then the trace representation of C is the code C_D of (1), where Tr is the trace function from $\text{GF}(2^3)$ to $\text{GF}(2)$.

Example 2 Let $q = 2$ and $n = 7$. Let C be the binary code with length $n = 7$ and generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \quad (12)$$

Then C has dimension $k = 4$. Let $m = \max\{k, \lceil \log_q n \rceil\} = 4$. Let α be a generator of $\text{GF}(2^4)^*$ with $\alpha^4 + \alpha + 1 = 0$. Define $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$, where

$$d_1 = 1, d_2 = \alpha, d_3 = \alpha^2, d_4 = \alpha^3, d_5 = \alpha^8, d_6 = \alpha^7, d_7 = \alpha^{11}.$$

Then the trace representation of C is the code C_D of (1), where Tr is the trace function from $\text{GF}(2^4)$ to $\text{GF}(2)$.

The trace representation of this section gives naturally a trace construction of all cyclic codes over finite fields without the condition that $\gcd(n, q) = 1$. Recall that Theorem 1 requires that $\gcd(n, q) = 1$.

4 Another trace representation of all cyclic codes over finite fields

In this section, we give a trace representation of all cyclic codes over finite fields. This representation may be related to the q -polynomial approach to cyclic codes developed in [6].

Let C be a cyclic code of length n over $\text{GF}(q)$. A polynomial

$$f(x) = \sum_{i=0}^{n-1} f_i x^i \in \text{GF}(q)[x]$$

generates C if and only if $\gcd(f(x), x^n - 1)$ is equal to the generator polynomial of C . There are many such polynomials $f(x)$ that generates C , e.g., the generator polynomial and the generating idempotent of C . Such a polynomial f can be employed to give a special trace representation of the code C . The following theorem gives such a representation.

Theorem 2 *Let $f(x) = \sum_{i=0}^{n-1} f_i x^i \in \text{GF}(q)[x]$ be any polynomial that generates a cyclic code C of length n over $\text{GF}(q)$. Let α be a normal element of $\text{GF}(q^n)$ over $\text{GF}(q)$. Define*

$$d = \sum_{j=1}^n f_{n-j} \alpha^{q^j}$$

and

$$D = \{d, d^q, d^{q^2}, \dots, d^{q^{n-1}}\}.$$

Then $C = C_D$, which is the trace code with defining set D , where the trace function is from $\text{GF}(q^n)$ to $\text{GF}(q)$.

Proof Since f is a generator polynomial of the cyclic code C , C has the following generator matrix:

$$\begin{bmatrix} f_0 & f_1 & f_2 & f_3 & \cdots & f_{n-3} & f_{n-2} & f_{n-1} \\ f_{n-1} & f_0 & f_1 & f_2 & \cdots & f_{n-3} & f_{n-1} & f_{n-2} \\ f_{n-2} & f_{n-1} & f_0 & f_1 & \cdots & f_{n-5} & f_{n-4} & f_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ f_2 & f_3 & f_4 & f_5 & \cdots & f_{n-1} & f_0 & f_1 \\ f_1 & f_2 & f_3 & f_4 & \cdots & f_{n-2} & f_{n-1} & f_0 \end{bmatrix}.$$

Since α is a normal element of $\text{GF}(q^n)$ over $\text{GF}(q)$, $\{\alpha, \alpha^q, \dots, \alpha^{q^{n-1}}\}$ is a normal basis of $\text{GF}(q^n)$ over $\text{GF}(q)$. Let

$$d_i = \sum_{j=0}^{n-1} f_{(j+i-1) \bmod n} \alpha^{q^j}.$$

for all i with $1 \leq i \leq n$. Then $d_i = d^{q^{i-1}}$ for all i . It then follows from the discussion in Section 3 that $C = C_D$.

Example 3 Let $q = 2$ and $n = 7$. Let C be the binary cyclic code with length $n = 7$ and generator polynomial $f(x) = 1 + x + x^3$. Then C has dimension $k = 4$. Let β be a generator of $\text{GF}(2^7)^*$ with $\beta^7 + \beta + 1 = 0$. Then $\alpha = \beta^{70}$ is a normal element of $\text{GF}(2^7)$. Then

$$d = \alpha^{2^6} + \alpha^{2^5} + \alpha^{2^3} = \beta^{14}.$$

Define $D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$, where

$$\begin{aligned} d_1 &= d^{q^0} = \beta^{14}, \\ d_2 &= d^{q^1} = \beta^{28}, \\ d_3 &= d^{q^2} = \beta^{56}, \\ d_4 &= d^{q^3} = \beta^{112}, \\ d_5 &= d^{q^4} = \beta^{97}, \\ d_6 &= d^{q^5} = \beta^{67}, \\ d_7 &= d^{q^6} = \beta^7. \end{aligned}$$

Then the trace representation of C is the code C_D of (1), where Tr is the trace function from $\text{GF}(2^7)$ to $\text{GF}(2)$.

The trace representation of Theorem 1 works under the condition that $\gcd(q, n) = 1$ only. Hence, it does not apply to all cyclic codes over finite fields. The advantage of Theorem 2 is that it applies to all linear codes. But its disadvantage is that the trace function is from a large extension field $\text{GF}(q^n)$ to $\text{GF}(q)$. The trace representation of all linear codes over finite fields gives automatically a trace representation of all cyclic codes over finite fields.

5 Summary and concluding remarks

The main contribution of this paper is the trace representation of all linear codes over finite fields described in Section 3. Consequently, any linear code C over a finite field may be generated with a definition set D via the trace construction. Hence, all types of linear codes over finite fields, including all cyclic codes, have a trace representation without having any restriction. This proves the claim in [3] and confirms that the defining-set method is indeed a fundamental construction of all linear codes over finite fields.

The trace construction of linear codes C_D has the following advantages over the generator matrix construction.

1. The description of the code C_D is simpler.
2. The determination of the weight distribution of C_D is much easier using the trace construction, as the weights of the codewords are expressed as character sums of the form (9).
3. It is possible to develop lower bounds on the minimum distance of linear codes when they are given as a trace code [9].

It is observed that in almost all cases of the determination of the weight distribution of linear codes, the trace construction has been employed.

A linear code over a finite field has many trace representations. Special types of linear codes may have special forms of trace representations. A trace representation of quasi-cyclic codes was given in [15]. A trace representation of quasi-negacyclic codes was presented in [14]. It would be interesting to develop other trace representations for special subclasses of linear codes.

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